Exam Special Relativity
January 21, 2014
Start: 18:30h End: 20:30

INSTRUCTIONS: This is a closed-book and closed-notes exam. The exam duration is 2 hours. You are allowed to use a numerical (non programmable) calculator. There is a total of 9 points that you can collect. Problems are designed, as much as it is possible, so that you can answer a given part of the problem without necessarily answer other parts. Work by default with SR units.

NOTE: If you are not asked to **Show your work**, then an answer is sufficient. However, you might always earn more points by answering more extensively (but you can also loose points by adding wrong explanations). If you are asked to **Show your work**, then you should explain your reasoning and the mathematical steps of your derivation in full. Use the official exam paper for *all* your work and ask for more if you need. Write clearly, and draw clearly the spacetime diagrams making use of the squared paper. **Solve the problems on separate sheets (1 sheet has four sides)**, i.e. problem 1 on sheet 1, problem 2 on sheet 2.

USEFUL FORMULAS AND CONSTANTS

$3.00 \times 10^8\,\mathrm{m} = 1\mathrm{s}$ $\qquad (1+x)^n \sim 1 + nx \quad x \ll 1$

$m^2 = E^2 - |\vec{P}|^2 = E^2 - P_x^2 - P_y^2 - P_z^2 \qquad P_x = \dfrac{mv_x}{\sqrt{1-v^2}} \qquad E = \dfrac{m}{\sqrt{1-v^2}}$

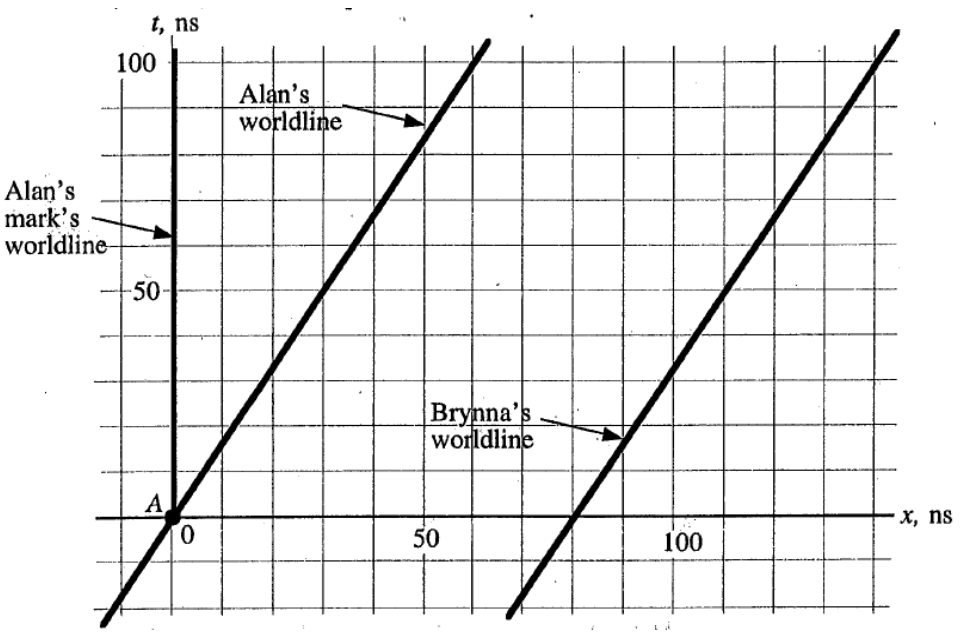
$\Delta t' = \gamma \Delta t - \gamma\beta\Delta x \qquad \gamma \equiv \dfrac{1}{\sqrt{1-\beta^2}}$

$\Delta x' = \gamma \Delta x - \gamma\beta\Delta t$

$\Delta y' = \Delta y$

$\Delta z' = \Delta z$

1. (5 points total) Alan and Brynna are passengers on a train that is moving through a long straight tunnel at a speed of $v = 3/5$ in a direction we will call the $+x$ direction. Alan and Brynna are 80 ns of distance apart as measured in the tunnel (Home) frame. At a pre-arranged instant of time in the train frame that we will call $t' = 0$, Alan reaches through his window and paints a mark on the tunnel wall (event A) and Brynna also reaches through her window and paints a mark on the wall (event B). Let Alan's position define $x' = 0$ in the train frame.



a) (2 points) In the diagram above, **draw and label** the x' axis for the train frame and **label** the t' axis also. **Calibrate** these axes as carefully as you can. Also, **draw event B** on the diagram above and **draw and label** the worldline of Brynna's mark.

b) (0.5 points) What is the value of the displacement x between the marks in the tunnel frame? Read the displacement directly from your diagram. **Explain your method** briefly.

c) (1 points) Check the value of x that you just found using an appropriate Lorentz transformation equation. **Show your work** [*Hint*: You should find $x > 100$ ns].

d) (1.5 points) How is it possible for Alan and Brynna, who are 80 ns apart in the tunnel frame to paint their marks simultaneously and yet leave marks on the tunnel that are more than 100 ns apart? **Explain your thinking.** [Don't just write something like "The events are not simultaneous" in one or the other frame. You get maximum credit for explaining exactly how a tunnel observer would explain what happens].

2. (4 points total) A particle is moving at constant speed along the $+x$ direction of the laboratory (Home) frame. Assume that the Home frame is inertial.

a) (2 points) If the particle speed is $v = 1/2$ in the Home frame (SR units), what is the velocity of the particle in a frame (Other frame) that moves at constant speed $\beta = 2/3$ along the $+x$ direction of the laboratory frame? **Show your work**: this means that you should also explicitly derive the relation between the velocities in the Home and the Other frame [*Hint*: recall that $v_x = dx/dt$ and $v'_x = dx'/dt'$].

b) (2 points) Consider now the same particle moving at constant speed $v = 1$ along the $+x$ direction of the Home frame. What is the velocity of the particle in the Other frame? **Show your work** and give a physical intepretation of the result.